

Fraction of the gluonium component in η' and η

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Abstract

It is interesting to determine the fraction of the gluonium component in η and η' which has been under serious discussion for many years. Measurements on different decay and/or production modes were employed in literatures, thus larger uncertainties were unavoidable. In this paper we suggest to determine the mixing angles of $\eta - \eta' - G$ using the data of semileptonic decays of D and D_s . We extract the mixing angles ϕ' , ϕ_G and the model parameters simultaneously. Thanks to the new measurements carried out by CLEO Collaboration and there are sufficient decay modes to determine both the model parameters and mixing angles. The mixing angles from data are $\phi' = (41.5 \pm 2.0)^\circ$ and $\sin \phi_G = 0.00 \pm 0.36$. Even though the central value of $\sin \phi_G$ is still zero, an upper bound is set. Moreover, as suggested in literature, $\eta(1405)$ is a glueball candidate whereas in our picture, $\eta(1405)$ may be identified as G with glueonium being its main content. Using all the model-parameters obtained above, we estimate the branching ratios of $D_s^+(D^+) \rightarrow Ge^+\nu_e$ where G is identified as $\eta(1405)$.

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I. INTRODUCTION

For a long time the mixing of $\eta - \eta'$ is of a great theoretical interest because it concerns many aspects of the underlying dynamic and the structure of pseudoscalar meson. One can investigate this mixing from two distinct schemes, namely they may be a mixture of the octet and singlet of the flavor SU(3), or $\frac{\sqrt{2}}{2}[\bar{u}u + \bar{d}d]$ and $\bar{s}s$ that are the mass eigenstates if we can assume an u-d degeneracy. The two schemes reflect different understandings of the essential physics. Moreover, the general principle of QCD implies that a gluonic degree of freedom of 0^{-+} may be involved in the physical states η and η' [1–11]. The QCD anomaly indeed may induce such a mixing. It would be crucially interesting to make a definite judgement if there is a sizable gluonic component in η and η' from either an underlying theory or phenomenology. Obviously even though the QCD anomaly is well formulated, it is hard to find its contribution to physical eigenstates because the matrix elements of the anomaly operator are dominated by the non-perturbative QCD and not reliably calculable so far. Therefore, one should turn to the phenomenological studies from which one may gain much information about the non-perturbative QCD, i.e. the mysterious aspect of the successful theory. In this work, we are going to determine the fraction of the glueball component in η and η' along the line.

Some researches[12–14] were done on this topic. Generally the authors obtained the mixing from different decay or production processes [15]. It can be conjectured that different modes are governed by different physics mechanisms besides the mixing which is the goal of our investigation, the uncertainties are not controllable, therefore the errors would be large. Instead, we suggest to use the data of semileptonic decays of D and D_s where the reaction mechanism should be the unique. Thus one can expect that in such a way, uncertainty could be greatly reduced. In the theoretical calculations, we employ the light-front-quark model (LFQM) which has been widely used for calculating the hadronic matrix elements. In the model, there are two independent model parameters (see the text for details) which are related to non-perturbative QCD and should be fixed by fitting data.

In our previous work [16], with the data of three independent measurements on $\mathcal{BR}(D^+ \rightarrow \eta e^+ \nu_e)$ [17], $\mathcal{BR}(D_s \rightarrow \eta e^+ \nu_e)$ and $\mathcal{BR}(D_s \rightarrow \eta' e^+ \nu_e)$ [18] we determined the $\eta - \eta'$ mixing $\phi = (39.9 \pm 2.6(exp) \pm 2.3(the))^\circ$ and predicted the branching ratio $\mathcal{BR}(D^+ \rightarrow \eta' e^+ \nu_e) = (2.12 \pm 0.23(exp) \pm 0.20(the)) \times 10^{-4}$ [16] which is consistent with the current data $\mathcal{BR}^{exp}(D^+ \rightarrow \eta' e^+ \nu_e) = (2.16 \pm 0.53 \pm 0.07) \times 10^{-4}$ measured by CLEO collaborator recently[19]. It is noted that there are three free parameters if the gluonic degree of freedom in η and η' is not accounted, namely two are the model parameters and another is the mixing angle. Instead, if one needs to consider the extra mixing between quark states with gluonic degree of freedom, there are four free parameters and at least four independent measurements are necessary.

Even though in the framework the assumption of null component of gluonium state is adopted and the predicted branching ratio of $\mathcal{BR}(D^+ \rightarrow \eta' e^+ \nu_e)$ is consistent with data, one

still cannot confirm that there indeed is no gluonium component in η and η' because the experimental errors are relatively large. Considering the present data, one can conclude that the central value of the fraction of gluonium in η and η' is consistent with zero, but while taking into account the error tolerance, instead, we would only be able to obtain an upper bound of its fraction. Fortunately, thanks to the new measurements carried out by the CLEO collaboration[19], the data for all the four decay modes are available, which enable us to simultaneously fit the two model parameters and the two mixing angles. Thus we may determine the fraction of the gluonium component in η and η' (concretely the upper bound of the gluonium component).

After this introduction, we describe the working framework and provide all necessary formulas. In the following section, we present our numerical results and the last section is devoted to the discussions and conclusion.

II. THE MODEL CALCULATION

In the theoretical framework where gluonium component is not accounted, the mixing matrix of $\eta - \eta'$ is set as [15, 20–22]

$$\begin{pmatrix} \eta \\ \eta' \end{pmatrix} = \begin{pmatrix} \cos \phi & -\sin \phi \\ \sin \phi & \cos \phi \end{pmatrix} \begin{pmatrix} \eta_q \\ \eta_s \end{pmatrix}, \quad (1)$$

where $\eta_q = \frac{1}{\sqrt{2}}(u\bar{u} + d\bar{d})$ and $\eta_s = s\bar{s}$ and it is noted that both of them are not SU(3) singlet. We referred this mixing as scenario-I in this work.

Since the QCD anomaly causes the mixing between η and η' , there is no any rule to forbid a mixing between the quark states and a glueball state of the quantum number 0^{-+} .

As one extends the picture to involve a gluonium component, a new scenario which we refer as the scenario-II, was suggested in Refs. [1–9] as

$$\begin{pmatrix} \eta \\ \eta' \\ G \end{pmatrix} = \begin{pmatrix} \cos \phi' & -\sin \phi' & 0 \\ \sin \phi' \cos \phi_G & \cos \phi' \cos \phi_G & \sin \phi_G \\ -\sin \phi' \sin \phi_G & -\cos \phi' \sin \phi_G & \cos \phi_G \end{pmatrix} \begin{pmatrix} \eta_q \\ \eta_s \\ g \end{pmatrix}, \quad (2)$$

where $|g\rangle = |\text{gluonium}\rangle$ is a pure gluonium state and the physical state G was identified as $\eta(1405)$ [6].

As discussed in the introduction, once there are data on $D^+ \rightarrow \eta e^+ \nu_e$ available in addition to the other three modes, we will be able to extract ϕ_G directly. Moreover new $\mathcal{BR}(D^+ \rightarrow \eta e^+ \nu_e) = (11.4 \pm 0.9 \pm 0.4) \times 10^{-4}$ deviates from the old datum and the combined branching ratios of $\mathcal{BR}(D_s \rightarrow \eta(\eta') e^+ \nu_e)$ [23] demands a redetermination of the mixing angles in (2).

Our strategy is that the mixing angles ϕ' , ϕ_G and the model parameters $\beta_{\eta(\eta')}^q$, $\beta_{\eta(\eta')}^s$ in the wave functions of η and η' are simultaneously determined by fitting solely one type of

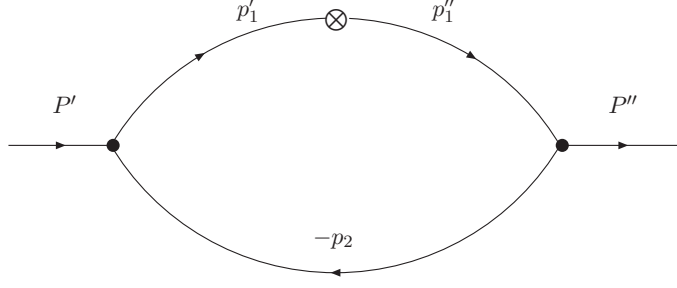


FIG. 1: Feynman diagram for meson transition amplitude

data i.e. we theoretically calculate the branching ratios of $D^+ \rightarrow \eta e^+ \nu_e$, $D^+ \rightarrow \eta' e^+ \nu_e$, $D_s \rightarrow \eta e^+ \nu_e$ and $D_s \rightarrow \eta' e^+ \nu_e$ and match them with data. Thus we “extract” the mixing angles directly from the semileptonic decays of D and D_s .

We use the light front quark model (LFQM) to evaluate the hadronic transition matrix elements [24–34], then obtain the decay widths of $D^+ \rightarrow \eta(\eta') e^+ \nu_e$ and $D_s \rightarrow \eta(\eta') e^+ \nu_e$ which are functions of ϕ' , ϕ_G , $\beta_{\eta(\eta')}^q$ and $\beta_{\eta(\eta')}^s$. The transition diagram is shown in Fig. 1.

The hadronic matrix elements for $P \rightarrow P$ transition can be parameterized as

$$\langle P(P'') | A_\mu | P(P') \rangle = f_+(q^2) \mathcal{P}_\mu + f_-(q^2) q_\mu. \quad (3)$$

where P represents the pseudoscalar meson, $P'(P'')$ is the momentum of initial (final) meson, $q = P' - P''$ and $\mathcal{P} = P' - P''$.

Functions $f_\pm(q^2)$ can be calculated in the LFQM and their explicit expressions were presented as [25],

$$\begin{aligned} f_+(q^2) &= \frac{N_c}{16\pi^3} \int dx_2 d^2 p'_\perp \frac{h'_p h''_p}{x_2 \hat{N}'_1 \hat{N}''_1} \left[-x_1 (M_0'^2 + M_0''^2) - x_2 q^2 + x_2 (m'_1 - m_1''^2) \right. \\ &\quad \left. + x_1 (m'_1 - m_2)^2 + x_1 (-m_1'' + m_2)^2 \right], \\ f_-(q^2) &= \frac{N_c}{16\pi^3} \int dx_2 d^2 p'_\perp \frac{2h'_p h''_p}{x_2 \hat{N}'_1 \hat{N}''_1} \left\{ x_1 x_2 M'^2 + p'_\perp + m'_1 m_2 + (-m_1'' + m_2)(x_2 m'_1 + x_1 m_2) \right. \\ &\quad \left. - 2 \frac{q \cdot \mathcal{P}}{q^2} \left(p_\perp'^2 + 2 \frac{(p'_\perp \cdot q_\perp)^2}{q^2} \right) - 2 \frac{(p'_\perp \cdot q_\perp)^2}{q^2} + \frac{(p'_\perp \cdot q_\perp)}{q^2} \left[M''^2 - x_2 (q^2 + q \cdot \mathcal{P}) \right. \right. \\ &\quad \left. \left. - (x_2 - x_1) M'^2 + 2x_1 M_0'^2 - 2(m'_1 - m_2)(m'_1 + m_1'') \right] \right\}. \end{aligned} \quad (4)$$

where m'_1 , m_1'' and m_2 are the corresponding quark masses, M' and M'' are the masses of the initial and final mesons respectively. All other notations can be found in the Appendix.

Since the calculations are done in space-like region one need to analytically continue them to the time-like region. It is convenient to redefine the matrix elements as

$$\langle P(P'') | A_\mu | P(P') \rangle = \left(\mathcal{P}_\mu - \frac{M'^2 - M''^2}{q^2} q_\mu \right) F_1(q^2) + \frac{M'^2 - M''^2}{q^2} q_\mu F_0(q^2). \quad (5)$$

The relations among the form factors are

$$F_1(q^2) = f_+(q^2), \quad F_0(q^2) = f_+(q^2) + \frac{q^2}{q \cdot \mathcal{P}} f_-(q^2). \quad (6)$$

The form factors in the space-like region are parameterized in a three-parameter form as

$$F(q^2) = \frac{F(0)}{1 - a \left(\frac{q^2}{M^2} \right) + b \left(\frac{q^2}{M^2} \right)^2}. \quad (7)$$

where F represents the form factor F_1 and F_0 . The parameters a , b and $F(0)$ are fixed by performing a three-parameter fit to the form factors in the space-like region. We then use these parameters to determine the physical form factors in the time-like region.

The input quark masses are directly taken from Ref. [25] as $m_u = 0.26$ GeV, $m_s = 0.37$ GeV, $m_c = 1.4$ GeV.

We first need to determine the model parameters for D and D_s . They can be fully determined by fitting their decay constants which are related to their total widths. For a pseudoscalar meson the decay constant can be evaluated

$$f_P = \frac{\sqrt{N_c}}{16\pi^3} \int dx_2 d^2 p'_\perp \frac{\varphi'}{\sqrt{2x_1 x_2} \tilde{M}'_0} 4(m'_1 x_2 + m_2 x_1). \quad (8)$$

The decay constants are experimentally measured as $f_D^{\text{exp}} = 0.221$ MeV, $f_{D_s}^{\text{exp}} = 0.27$ MeV. With them the model parameters for β_{D_s} and β_D are fixed to be $\beta_D = 0.499$ GeV, $\beta_{D_s} = 0.592$ GeV [35]. Since the total widths are better measured, the model parameters of the heavy mesons D and D_s are determined with higher accuracy.

Once the model parameters at the initial side are fixed, we would turn to concern the decay products. At first look, there are six free parameters β_η^q , $\beta_{\eta'}^q$, β_η^s , $\beta_{\eta'}^s$, ϕ' and ϕ_G to be fixed. It seems that there are not enough equations to determine all these parameters. However, as discussed in Ref.[16] two relations $\beta_\eta^q = \beta_{\eta'}^q$ and $\beta_\eta^s = \beta_{\eta'}^s$ hold, thus the number of unknowns reduces to four. Matching the theoretically calculated branching ratios with the data for the four decay modes, we obtain all the values of ϕ' , ϕ_G , $\beta_{\eta(\eta')}^q$, $\beta_{\eta(\eta')}^s$. The mixing angles are fitted to be $\phi' = (41.5 \pm 2.0)^\circ$ and $\sin \phi_G = 0.00 \pm 0.36$ ($\sin^2 \phi_G = 0.00 \sim 0.13$) where the errors are from the experimental side. The model parameters are $\beta_{\eta(\eta')}^q = 0.35$ GeV and $\beta_{\eta(\eta')}^s = 0.59$ GeV.

With these parameters, assuming $\eta(1405)$ to be the physical state G whose main content is glueonium g , we are able to estimate the branching ratios of D_s decaying into eta(1405) via its $q\bar{q}$ components and we obtain $\mathcal{BR}(D_s^+ \rightarrow Ge^+\nu_e) = 0 \sim 8.6 \times 10^{-4}$ and $\mathcal{BR}(D^+ \rightarrow Ge^+\nu_e) = 0 \sim 1.1 \times 10^{-5}$. Accurate measurements on these semi-leptonic decay modes may tell us if the main content of $\eta(1405)$ is glueonium (i.e. a glueball candidate) and further test the Scenario II for the mixing.

III. DISCUSSION

In Ref.[9] by fitting the data of several radiative decay modes such as $\omega \rightarrow \eta\gamma$, $\rho \rightarrow \eta\gamma$ and $\omega \rightarrow \pi^0\gamma$ $\sin^2\phi_G = 0.115 \pm 0.036$ ¹ and $\phi' = (40.4 \pm 0.6)^\circ$ was fixed if one sets $\sin^2\phi_G$ as a free parameter.

The value of ϕ' obtained from different kinds of experiments are consistent with each other in a reasonable error tolerance but the central value of $\sin^2\phi_G$ diverges over a range. The central value of $\sin^2\phi_G$ obtained from the semileptonic decays of D and D_s inclines to that the fraction of the gluonium state in η and η' is consistent with zero, but since there exists a relatively large uncertainty in the data, it is hard to conclude that it does not exist, i.e. $\sin^2\phi_G = 0$. It is urgent to improve the precision of measurement on these channels especially on $D(D_s) \rightarrow \eta' e \nu_e$.

In this work, we employ the LFQM model to evaluate the hadronic transition matrix elements. We fix the model parameters by fitting data, so that some theoretical uncertainties are involved in those parameters. Definitely the results are still model dependent because we employ a concrete model: the LFQM. But since all inputs are taken from the same source (i.e. the data of the semi-leptonic decays of D and D_s), one can expect that relative errors would be partly compensated, so that the model-dependence of the results is somehow alleviated.

Because of absence of the final state interactions, the semileptonic decays have obvious advantages for determining the properties of the produced light hadrons, such as the structure of $f_0(980)$, $\eta - \eta'$ mixing and even a mixing of pseudoscalar mesons with glueball.

Moreover, as suggested, assuming that the physical state $\eta(1405)$ is the mixture G whose main content is glueonium, we calculate the branching ratios of $\mathcal{BR}(D_s^+ \rightarrow Ge^+\nu_e)$ and $\mathcal{BR}(D^+ \rightarrow Ge^+\nu_e)$. The semileptonic decay modes will be measured by the future experiments. Then the proposed scenario II for the mixing of $q\bar{q}$, $s\bar{s}$ and glueonium g will be tested.

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¹ the error comes from the χ^2 method.

Appendix A

Here we list some variables appearing in the context. The incoming (outgoing) meson in Fig. 1 has the momentum $P'^{(n)} = p_1'^{(n)} + p_2$ where $p_1'^{(n)}$ and p_2 are the momenta of the off-shell quark and antiquark and

$$\begin{aligned} p_1'^+ &= x_1 P'^+, & p_2^+ &= x_2 P'^+, \\ p_{1\perp}' &= x_1 P_{\perp}' + p_{\perp}', & p_{2\perp} &= x_2 P_{\perp}' - p_{\perp}', \end{aligned} \quad (\text{A1})$$

with x_i and p_{\perp}' are internal variables and $x_1 + x_2 = 1$.

The variables M_0' , \tilde{M}_0' , h_p' and \hat{N}_1' are defined as

$$\begin{aligned} M_0'^2 &= \frac{p_{\perp}'^2 + m_1'^2}{x_1} + \frac{p_{\perp}'^2 + m_2^2}{x_2}, \\ \tilde{M}_0' &= \sqrt{M_0'^2 - (m_1' - m_2)^2}. \end{aligned} \quad (\text{A2})$$

$$h_p' = (M'^2 - M_0'^2) \sqrt{\frac{x_1 x_2}{N_c}} \frac{1}{\sqrt{2} \tilde{M}_0'} \varphi', \quad (\text{A3})$$

where

$$\varphi' = 4 \left(\frac{\pi}{\beta'^2} \right)^{3/4} \sqrt{\frac{dp_z'}{dx_2}} \exp\left(-\frac{p_z'^2 + p_{\perp}'^2}{2\beta'^2}\right), \quad (\text{A4})$$

with $p_z' = \frac{x_2 M_0'}{2} - \frac{m_2^2 + p_{\perp}'^2}{2x_2 M_0'}$.

$$\hat{N}_1' = x_1 (M'^2 - M_0'^2). \quad (\text{A5})$$

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- [1] J. Rosner, Phys. Rev. **D 27**, 1101 (1983);
 - [2] P. Ball, J. M. Frere and M. Tytgat, Phys. Lett. B **365**, 367 (1996) [arXiv:hep-ph/9508359].
 - [3] M. Benayoun, L. DelBuono, S. Eidelman, V. N. Ivanchenko and H. B. O'Connell, Phys. Rev. D **59**, 114027 (1999) [arXiv:hep-ph/9902326].
 - [4] F. J. Gilman and R. Kauffman, Phys. Rev. D **36**, 2761 (1987).
 - [5] A. Bramon, R. Escribano and M. D. Scadron, Phys. Lett. B **503**, 271 (2001).
 - [6] H. Y. Cheng, H. N. Li and K. F. Liu, Phys. Rev. D **79**, 014024 (2009).
 - [7] F. Ambrosino *et al.* [KLOE Collaboration], Phys. Lett. B **648**, 267 (2007).
 - [8] V. Mathieu and V. Vento, arXiv:0910.0212 [hep-ph].
 - [9] F. Ambrosino *et al.*, JHEP **0907**, 105 (2009) [arXiv:0906.3819 [hep-ph]].

- [10] C. E. Thomas, JHEP **0710**, 026 (2007) [arXiv:0705.1500 [hep-ph]].
- [11] R. Escribano and J. Nadal, JHEP **0705**, 006 (2007) [arXiv:hep-ph/0703187].
- [12] S. S. Gershtein and M. Y. Khlopov, JETP Lett. **23**, 338 (1976).
- [13] K. T. Chao, Nucl. Phys. B **317**, 597 (1989); Nucl. Phys. B **335**, 101 (1990).
- [14] C. Degrande and J. M. Gerard, JHEP **0905**, 043 (2009) [arXiv:0901.2860 [hep-ph]].
- [15] T. Feldmann, P. Kroll and B. Stech, Phys. Rev. D **58**, 114006 (1998) [arXiv:hep-ph/9802409].
- [16] H. W. Ke, X. Q. Li and Z. T. Wei, Eur. Phys. J. C **69**, 133 (2010) [arXiv:0912.4094 [hep-ph]].
- [17] R. E. Mitchell *et al.* [CLEO Collaboration], Phys. Rev. Lett. **102**, 081801 (2009) [arXiv:0802.4222 [hep-ex]].
- [18] J. Yelton *et al.* [CLEO Collaboration], arXiv:0903.0601 [hep-ex].
- [19] J. Yelton *et al.* [CLEO Collaboration], arXiv:1011.1195 [hep-ex].
- [20] D. Kekez, D. Klabužar and M. Scadron, J. Phys. **G 27**, 1775 (2001).
- [21] M. Scadron, Phys. Rev. D **26**, 239 (1982).
- [22] S. Fajfer and J. F. Kamenik, Phys. Rev. D **71**, 014020 (2005) [arXiv:hep-ph/0412140].
- [23] K. Nakamura *et al.* [Particle Data Group], J. Phys. G **37**, 075021 (2010).
- [24] W. Jaus, Phys. Rev. D **60**, 054026 (1999).
- [25] H. Y. Cheng, C. K. Chua and C. W. Hwang, Phys. Rev. D **69**, 074025 (2004).
- [26] C. W. Hwang and Z. T. Wei, J. Phys. G **34**, 687 (2007).
- [27] C. D. Lu, W. Wang and Z. T. Wei, Phys. Rev. D **76**, 014013 (2007) [arXiv:hep-ph/0701265].
- [28] H. W. Ke, X. Q. Li and Z. T. Wei, Phys. Rev. D **77**, 014020 (2008).
- [29] Z. T. Wei, H. W. Ke and X. Q. Li, Phys. Rev. D **80**, 094016 (2009) [arXiv:0909.0100 [hep-ph]].
- [30] H. W. Ke, X. Q. Li and Z. T. Wei, Phys. Rev. D **80**, 074030 (2009) [arXiv:0907.5465 [hep-ph]].
- [31] H. W. Ke, X. Q. Li, Z. T. Wei and X. Liu, Phys. Rev. D **82**, 034023 (2010) [arXiv:1006.1091 [hep-ph]].
- [32] H. W. Ke, X. Q. Li and X. Liu, arXiv:1002.1187 [hep-ph].
- [33] G. Li, F. I. Shao and W. Wang, Phys. Rev. D **82**, 094031 (2010) [arXiv:1008.3696 [hep-ph]].
- [34] P. Colangelo, F. De Fazio and W. Wang, Phys. Rev. D **81**, 074001 (2010) [arXiv:1002.2880 [hep-ph]].
- [35] Z. T. Wei, H. W. Ke and X. F. Yang, Phys. Rev. D **80**, 015022 (2009) [arXiv:0905.3069 [hep-ph]].